Extra Homework Assignments

Bifurcations: Theory and Applications

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Problem 1: Let Ψ_1 and Ψ_2 denote smooth diffeomorphisms of $\mathbb{R}^N$ and $A, B \in \mathbb{R}^{N\times N}$. Show the following claims:

(i) For the pullback $\Psi^* f := (D\Psi)^{-1} \circ f \circ \Psi$ the composition satisfies

$$(\Psi_2 \circ \Psi_1)^* = \Psi_1^* \Psi_2^*.$$

(ii) For the adjoint $\mathrm{ad}Af(x) := Af(x) - f'(x)Ax$, the matrix commutator $[A, B] := AB - BA$ satisfies

$$[\mathrm{ad}A, \mathrm{ad}B] = \mathrm{ad}[A, B].$$

(iii) For $A[\Psi](x) := A\Psi(x) - \Psi(Ax)$, the matrix commutator satisfies

$$[A[\cdot], B[\cdot]] = [A, B][\cdot].$$

Problem 2: Let $\Phi_t(f)$ denote the flow of a C^1 vector field

$$\dot{x} = f(x) \text{ such that } f(0) = 0,\ x \in \mathbb{R}^N.$$

Use standard theorems on differentiation of flows with respect to initial conditions to show

$$D_x(\Phi_t(f))(0) = \Phi_t(D_x f(0)).$$

Problem 3: Let B be a real 2×2 matrix with algebraically double eigenvalue -1. Suppose $B = \exp(A)$ for some real 2×2 matrix A. Prove or disprove

$$B = -\mathrm{id}\,.$$

Problem 4: For $N \in \mathbb{N}$, describe the Lie algebras, $\mathfrak{g}$, of the following Lie groups, $\mathcal{G}$, seen in the lecture:

(i) $SU(N)$,

(ii) $GL(N, \mathbb{C})$,

(iii) $SL(N, \mathbb{R})$.

Hint (iii): Note that, in our case, the usual matrix exponentiation yields a 1-parameter subgroup of $\mathcal{G}$ for all $\mathfrak{a} \in \mathfrak{g}$

$$\{\exp(\mathfrak{a}t) \mid t \in \mathbb{R}\}.$$